The Informational Factor of Production and the Systematic Mispricing of Personal Data Inputs

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Abstract

Modern firms increasingly depend on structured observations of economic agents in order to reduce uncertainty and improve decision-making. Transaction histories, behavioral signals, mobility traces, and other forms of personal data allow firms to forecast demand, optimize logistics, personalize services, and allocate resources more efficiently. Despite the central role of these informational inputs in modern production systems, standard economic theory continues to treat capital and labor as the only primary factors of production. This paper introduces informational stock—defined as measurable reductions in uncertainty about economically relevant outcomes—as an independent factor of production. Informational stock is formally defined using Shannon mutual information and incorporated into a neoclassical production framework. The model demonstrates that when informational inputs are omitted from factor pricing, their contribution is systematically attributed to capital income, generating an upward bias in measured capital shares. This misattribution implies that a substantial portion of modern economic output currently attributed to capital is in fact produced by informational inputs derived from personal data. Using stylized industry calibrations and market-level estimates, the paper argues that informational factor mispricing may represent one of the largest structural distortions in contemporary capital markets.

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1 Introduction

Economic production has historically been modeled as a combination of capital and labor. From the earliest formulations of classical political economy to the formalization of neoclassical growth theory, economists have treated output as the result of these two inputs interacting through a production technology. Within this framework, competitive equilibrium implies that factors of production are paid according to their marginal products. Capital earns returns proportional to its productivity, labor earns wages proportional to its productivity, and observed income shares reflect the underlying structure of production.

Over the past several decades, however, the structure of production in advanced economies has undergone a profound transformation. Firms increasingly operate in environments where productivity depends not only on physical equipment and human effort but also on the ability to collect, interpret, and act upon large volumes of information about economic agents. Digital platforms track consumer behavior in real time. Retailers forecast demand using detailed purchase histories. Logistics systems optimize routing based on millions of location observations. Financial institutions evaluate risk using predictive models trained on large datasets of past transactions. In each of these cases, productivity improvements arise not primarily from additional capital or labor, but from reductions in uncertainty enabled by structured observations of human activity.

These informational inputs have become central to the functioning of modern firms. Yet economic theory and measurement have not kept pace with this transformation. Standard production models typically embed the productivity effects of information within capital inputs, technological progress, or residual productivity terms. As a result, the economic contribution of informational inputs remains largely invisible in both theoretical and empirical frameworks.

The omission of informational inputs from production theory has important consequences. If information plays a productive role in modern firms, then failing to recognize it as an independent factor of production necessarily distorts measured factor shares. In particular, when informational productivity is omitted from production functions, its contribution must be absorbed by the remaining factors. Because informational systems are often embedded within capital-intensive infrastructures—such as cloud computing systems, data centers, and digital platforms—the productivity generated by informational inputs is frequently attributed to capital. This attribution inflates measured capital income shares and obscures the role that informational inputs play in generating economic output.

This paper proposes a framework for incorporating informational inputs directly into production theory. The central concept introduced in the paper is informational stock, defined as the measurable reduction in uncertainty about economically relevant outcomes produced by structured observations. Informational stock is formally grounded in information theory, specifically Shannon's concept of mutual information, which measures the degree to which one variable reduces uncertainty about another. Within the context of economic production, informational stock represents the predictive knowledge a firm possesses about outcomes such as consumer demand, asset risk, or logistical conditions.

Once informational stock is defined in this way, it can be incorporated into a production function alongside capital and labor. Firms combine capital equipment, human labor, and informational stock in order to generate output. Informational stock improves production by enabling more accurate forecasts, more efficient allocation of resources, and better coordination across complex economic systems.

Introducing informational stock into production theory yields a straightforward but powerful result. If informational inputs are productive but omitted from factor pricing, then the observed capital share of income must exceed the true capital share by the amount of informational productivity embedded in capital returns. In other words, informational factor income is statistically absorbed into capital income. This mechanism implies that modern estimates of capital productivity systematically overstate the role of capital in generating economic output.

The implications of this result are substantial. Over the past several decades, economists have documented a steady rise in the capital share of income across many advanced economies. Explanations for this phenomenon have ranged from technological change to market concentration and globalization. The framework proposed in this paper suggests an additional explanation: part of the observed rise in capital income may reflect the growing economic importance of informational inputs that remain unmeasured in production models.

The argument developed here is not merely conceptual. Informational stock can be measured empirically through improvements in predictive performance. When a dataset reduces uncertainty about an outcome variable, the reduction in predictive loss provides a quantitative measure of informational value. These improvements can be directly linked to production outcomes through their effects on pricing, allocation, and operational decisions. As a result, the informational factor introduced in this paper is not purely theoretical; it can, in principle, be estimated and incorporated into empirical production models.

To illustrate the magnitude of the potential distortion created by informational mispricing, the paper presents stylized industry calibrations based on publicly observable firm characteristics. In sectors such as mobility platforms, retail logistics, and airline operations, predictive systems built on large-scale behavioral datasets generate substantial productivity gains. When these gains are interpreted through the lens of informational stock, a significant portion of firm value appears attributable to informational inputs rather than traditional capital accumulation.

At the market level, the implications are even larger. Public companies in the United States collectively represent tens of trillions of dollars in market capitalization. If even a modest portion of that value reflects informational productivity currently attributed to capital, then the aggregate magnitude of informational factor mispricing could reach several trillion dollars.

This paper therefore advances a strong claim: informational inputs derived from personal data constitute a distinct factor of production, and the failure to recognize this factor has produced one of the largest systematic mismeasurements in modern economic accounting.

The remainder of the paper proceeds as follows. Section 2 reviews related literature in information theory, production economics, and the economics of intangible assets. Section 3 defines informational stock formally and establishes its properties as an economic input. Section 4 introduces informational stock into a neoclassical production framework and derives equilibrium factor pricing conditions. Section 5 demonstrates that omitting informational inputs from production estimation generates upward bias in measured capital shares. Section 6 develops a measurement framework for estimating informational stock using predictive performance improvements. Section 7 provides stylized industry simulations illustrating the magnitude of informational factor productivity in several sectors. Section 8 discusses implications for national income accounting and growth measurement. Section 9 addresses potential objections and identification challenges. Section 10 concludes.

2 Literature Review

The framework developed in this paper intersects with several strands of existing economic and technical research. These include the economics of information, the theory of production and economic growth, the measurement of intangible capital, and recent empirical work on data-driven productivity. While each of these literatures recognizes aspects of the growing economic importance of information, none provides a production framework in which reductions in uncertainty themselves constitute an explicitly priced factor of production. The present paper contributes to this gap by defining informational stock in measurable terms and incorporating it directly into the structure of production theory.

2.1 Information Theory and Economic Measurement

The concept of informational stock developed in this paper is grounded in information theory, particularly the work of Claude Shannon (1948). Shannon introduced entropy as a measure of uncertainty associated with a random variable and mutual information as the amount by which observing one variable reduces uncertainty about another. These concepts have since become foundational tools in statistics, signal processing, machine learning, and communication theory.

Entropy provides a quantitative measure of uncertainty, defined as the expected information content of a random variable. Mutual information measures the reduction in uncertainty achieved by observing an additional variable. In statistical learning and predictive modeling, these measures describe how much information a dataset provides about a target variable. In other words, they quantify the degree to which observations make outcomes more predictable.

Although information theory originated in the study of communication systems, its conceptual framework is naturally applicable to economic production. Firms constantly attempt to reduce uncertainty about economically relevant outcomes such as consumer demand, asset risk, supply chain disruptions, and market conditions. Data collection, predictive modeling, and algorithmic decision systems are all mechanisms through which firms acquire information about these outcomes. When these systems succeed, they improve predictions and allow firms to allocate resources more efficiently.

Despite this clear connection between uncertainty reduction and economic productivity, standard economic models rarely incorporate formal measures of information into production theory. Information is sometimes discussed conceptually in the economics of uncertainty or the theory of expectations, but the explicit use of information-theoretic quantities such as entropy or mutual information remains uncommon in production economics. This paper proposes that informational inputs measured through uncertainty reduction can be treated as a productive input alongside capital and labor.

2.2 Production Theory and Economic Growth

The canonical framework for analyzing production in modern economics is the neoclassical production function. Beginning with the work of Solow (1957), economists have modeled aggregate output as a function of capital and labor inputs combined through a production technology. Within this framework, economic growth arises from increases in capital accumulation, labor supply, or technological progress.

In the Solow model, technological progress appears as an exogenous factor that shifts the production function over time. Later work in endogenous growth theory, particularly Romer (1990), emphasized the role of knowledge accumulation and innovation as drivers of technological progress. In these models, knowledge functions as a non-rival input that can increase productivity across many production processes simultaneously.

The concept of informational stock proposed in this paper shares certain characteristics with knowledge in endogenous growth models. Both represent non-rival inputs that improve productivity without being depleted through use. However, informational stock differs from general knowledge in several important respects. First, informational stock is defined operationally in terms of uncertainty reduction about specific economic outcomes rather than abstract technological knowledge. Second, informational stock can be measured empirically using predictive performance metrics derived from statistical models. Third, informational stock often originates directly from observations of economic agents, particularly through the collection of behavioral and transactional data.

For these reasons, informational stock represents a more concrete and measurable form of productive knowledge than the abstract technology terms commonly used in growth theory.

2.3 Intangible Capital and Modern Firm Value

A separate but related literature examines the growing importance of intangible assets in modern economies. Corrado, Hulten, and Sichel (2009) document the increasing role of intangible capital in the United States and argue that investments in knowledge, software, brand equity, and organizational capital represent a substantial share of modern economic activity. Subsequent work has extended these findings and shown that intangible assets now account for a large portion of corporate value.

Financial studies of corporate balance sheets have similarly observed a dramatic shift in the composition of firm value. Research by Ocean Tomo and others has shown that intangible assets represent the majority of market value in many large publicly traded firms. These intangible assets include intellectual property, software, brand reputation, and other non-physical sources of economic value.

While this literature demonstrates that non-physical assets have become central to modern firms, it does not isolate informational inputs as a distinct factor of production. Intangible capital is typically measured as an aggregate category encompassing a wide range of assets, from patents to organizational knowledge. Informational stock as defined in this paper represents a more specific concept: the reduction of uncertainty about economic outcomes through structured observations.

Many modern intangible assets derive their value precisely because they enable firms to generate and exploit informational stock. Data collection systems, machine learning infrastructure, and digital platforms all produce value by converting observations into predictive knowledge. Treating informational stock as a separate factor of production therefore helps clarify the mechanism through which many intangible assets contribute to economic output.

2.4 Data-Driven Decision Making

A growing body of empirical work examines the productivity effects of data-driven decision making. Brynjolfsson, Hitt, and Kim (2011) provide evidence that firms using data-intensive management practices experience higher productivity and improved performance. Similar research has shown that firms that systematically collect and analyze data often outperform competitors in areas such as operational efficiency, marketing effectiveness, and logistics management.

These findings suggest that the ability to extract actionable insights from data can substantially increase firm productivity. However, empirical studies of data-driven decision making typically treat data as an input to management practices rather than as a formal factor of production. The present paper extends this literature by linking the productivity effects of data directly to reductions in uncertainty measured through information theory.

Within this framework, data-driven productivity improvements arise because observational datasets reduce uncertainty about economically relevant outcomes. When uncertainty decreases, predictions become more accurate and decisions become more efficient. Informational stock therefore serves as the mechanism through which data generates productivity gains.

2.5 Personal Data and Economic Production

An additional dimension of informational production arises from the role of personal data. Many modern datasets consist of observations generated by individuals through their economic and social activities. Examples include purchase histories, mobility traces, communication patterns, and online behavior. These observations provide valuable information about preferences, demand patterns, and behavioral responses.

When aggregated and analyzed, personal data allows firms to construct predictive models of consumer behavior, market demand, and operational conditions. These models often form the basis of algorithmic systems that allocate resources, recommend products, set prices, and coordinate complex supply chains. As a result, the economic productivity of many modern firms depends heavily on informational inputs derived from the activities of individuals.

Despite this central role, the economic contribution of personal data remains largely invisible in traditional factor accounting. Production models typically attribute productivity gains generated by predictive systems to capital or technology, even when those systems rely heavily on observational data derived from human activity.

The framework developed in this paper provides a way to recognize this contribution explicitly by treating informational stock derived from personal data as a productive input in its own right.

2.6 Contribution of the Present Paper

The central contribution of this paper is to integrate these various strands of research into a unified production framework. Drawing on information theory, the paper defines informational stock as the reduction in uncertainty about economically relevant outcomes produced by structured observations. This concept allows informational inputs to be incorporated into production functions alongside capital and labor.

The resulting model demonstrates that when informational inputs are productive but omitted from factor pricing, their contribution is absorbed into capital income shares. This mechanism provides a theoretical explanation for why capital appears increasingly productive in modern economies.

By linking information theory, production economics, and empirical observations of data-driven productivity, the paper provides a framework for measuring informational inputs and evaluating their role in modern economic production.

3 Conceptual Foundations and Definitions

The central objective of this paper is to introduce informational stock as a measurable factor of production. To do so, it is necessary to clarify how information enters economic production and how it can be represented formally within a production framework. The concept developed here draws directly from information theory, where information is defined as the reduction of uncertainty about a random variable. When applied to economic activity, this reduction in uncertainty improves the ability of firms to predict and respond to economically relevant outcomes.

In modern production systems, firms routinely confront uncertainty about variables such as consumer demand, transportation conditions, asset risk, and market behavior. The ability to reduce uncertainty about these variables allows firms to make better decisions about pricing, inventory, logistics, risk allocation, and resource deployment. Informational stock represents the cumulative predictive knowledge a firm possesses about these outcomes.

The concept of informational stock therefore reflects an operational definition of information: information is valuable to the extent that it reduces uncertainty about outcomes that matter for economic decisions.

3.1 Entropy and Economic Uncertainty

In information theory, uncertainty about a random variable is measured using Shannon entropy. Let Y denote a discrete random variable representing an economically relevant outcome. Examples include the demand for a product, the probability of an insurance claim, the arrival time of a shipment, or the likelihood of a transaction occurring.

The entropy of Y is defined as

$$H(Y) = -\sum_{y} p(y) \log\, p(y)$$

Entropy measures the expected uncertainty associated with the outcome of the variable. When the distribution of outcomes is highly unpredictable, entropy is large. When outcomes are highly predictable, entropy is small.

In economic environments, many outcomes of interest initially exhibit substantial uncertainty. Firms therefore devote significant resources to collecting and analyzing observations that allow them to reduce this uncertainty.

3.2 Observations and Conditional Uncertainty

Suppose a firm observes a set of variables X. These variables may include transaction histories, customer behavior, environmental conditions, operational signals, or other observational data relevant to predicting the outcome variable Y.

Once these observations are available, the uncertainty associated with Y may be reduced. The remaining uncertainty is represented by conditional entropy:

$$H(Y \mid X)$$

Conditional entropy measures the uncertainty that remains about the outcome variable once the observational variables are known.

If the observations contain no useful information about the outcome variable, then conditional entropy will be approximately equal to the original entropy. However, if the observations contain predictive signals, conditional entropy will be smaller than the original entropy.

3.3 Mutual Information

The reduction in uncertainty produced by the observations is measured by mutual information. Mutual information quantifies the degree to which observing X reduces uncertainty about Y. It is defined as

$$I(X;Y) = H(Y) - H(Y \mid X)$$

Mutual information therefore measures the amount of predictive knowledge contained in the observational variables.

When applied to economic production, mutual information represents the degree to which a firm's observational datasets improve its ability to predict economically relevant outcomes. Higher mutual information corresponds to more accurate predictions and more informed decisions.

For example, consider a retailer attempting to forecast product demand. Without observational data, the retailer may face substantial uncertainty about how many units of a product will sell in a given time period. However, by analyzing past purchases, browsing behavior, and seasonal patterns, the retailer may be able to reduce uncertainty about future demand. The reduction in uncertainty generated by these observations can be measured using mutual information.

3.4 Informational Stock

Building on these concepts, this paper defines informational stock as the cumulative predictive knowledge available to a firm about economically relevant outcomes. Formally, informational stock is defined as the mutual information between a firm's observational variables and the outcome variables relevant to its production process.

Let Zrepresent a transformation of the observational variables Xproduced by data processing systems, statistical models, or machine learning algorithms. These transformations may include feature engineering, predictive modeling, or other forms of data analysis.

Informational stock is therefore defined as

$$I = I(Z;Y)$$

where Zrepresents the processed informational signals used by the firm.

In this framework, informational stock represents the degree to which a firm's information systems reduce uncertainty about outcomes that influence economic decisions.

3.5 Economic Interpretation

The economic significance of informational stock arises from its effect on decision-making. When firms possess greater predictive knowledge about economically relevant variables, they can allocate resources more efficiently. Better predictions allow firms to reduce waste, avoid costly mistakes, and identify profitable opportunities that might otherwise remain hidden.

Several common examples illustrate this mechanism.

Retail firms use demand forecasting systems to determine how much inventory to stock at each location. Mobility platforms use location data and demand predictions to match drivers with passengers efficiently. Airlines use predictive models to optimize pricing and seat allocation across flights. Financial institutions use predictive systems to assess credit risk and allocate capital.

In each case, productivity improvements arise because informational systems reduce uncertainty about relevant outcomes. The resulting predictions enable firms to make more efficient decisions.

From the perspective of production theory, informational stock therefore functions as an input that enhances the productivity of both capital and labor. Capital equipment and human workers operate more efficiently when decisions are guided by accurate predictions. Informational stock therefore contributes directly to output by improving the allocation of resources throughout the production process.

3.6 Properties of Informational Stock

Informational stock exhibits several properties that distinguish it from traditional factors of production.

First, informational stock is largely non-rivalrous. Once information has been generated, it can often be used simultaneously across multiple production processes without being depleted. A predictive model developed for one application can frequently be applied to many transactions or decisions at minimal additional cost.

Second, informational stock can exhibit increasing returns through aggregation. When datasets grow larger and more diverse, predictive models may become more accurate. The accumulation of observations can therefore increase the informational productivity of existing data.

Third, informational stock is durable but not permanent. Information can remain useful over extended periods, but its predictive value may decline as economic conditions change. For this reason, informational systems often require continuous updating through the collection of new observations.

Finally, informational stock can be measured empirically through improvements in predictive performance. When observational datasets reduce predictive error in statistical models, the resulting improvement provides a quantifiable measure of informational value.

3.7 Informational Stock as a Factor of Production

These characteristics suggest that informational stock can be treated as an independent factor of production alongside capital and labor. Firms combine capital equipment, human labor, and informational stock to produce economic output. Informational stock improves productivity by reducing uncertainty and enabling more efficient decisions.

The next section formalizes this idea by introducing informational stock directly into a neoclassical production function and deriving the implications for factor pricing and income shares.

4 Informational Stock in the Production Function

Having defined informational stock as a measurable reduction in uncertainty about economically relevant outcomes, the next step is to incorporate this concept directly into a production framework. This section formalizes informational stock as an independent input in the production process and examines the implications for firm behavior and factor pricing.

Traditional neoclassical production theory models output as a function of capital and labor inputs. In its simplest form, output Y is represented as

$$Y = F(K, L)$$

where K denotes capital and L denotes labor. The production function summarizes the technology through which these inputs are transformed into output.

However, the analysis in the previous section suggests that informational inputs play a distinct role in modern production systems. Firms collect observations about their economic environment, transform these observations into predictive knowledge, and use that knowledge to improve decisions about pricing, allocation, and coordination. These informational resources allow firms to deploy capital and labor more efficiently than would otherwise be possible.

To capture this mechanism formally, the production function can be extended to include informational stock as an additional input:

$$Y = F(K, L, I)$$

where I represents informational stock as defined in the previous section.

In this formulation, output depends not only on the quantity of capital and labor available to the firm but also on the predictive knowledge the firm possesses about economically relevant outcomes. Informational stock improves productivity by enabling more efficient allocation of resources throughout the production process.

4.1 Assumptions

To analyze the implications of this extended production function, several standard assumptions are introduced.

First, the production function $F(K, L, I)$is assumed to be continuously differentiable with respect to each input. This assumption allows marginal productivity conditions to be defined.

Second, the marginal product of each input is assumed to be positive:

$$F_K > 0, F_L > 0, F_I > 0$$

These conditions reflect the idea that increasing any input while holding the others fixed increases output.

Third, the production function is assumed to exhibit constant returns to scale. Under this assumption, scaling all inputs by a common factor scales output by the same factor. Formally,

$$F(\lambda K, \lambda L, \lambda I) = \lambda F(K, L, I)$$

for any positive scalar λ.

Constant returns to scale is a common assumption in competitive production models because it ensures that firms operating in competitive markets earn zero economic profits in equilibrium.

4.2 Informational Productivity

Within this framework, informational stock affects output through its marginal productivity. The marginal product of informational stock is defined as

$$F_I = \frac{\partial F}{\partial I}$$

This quantity measures the increase in output that results from an additional unit of informational stock while holding capital and labor constant.

Economically, the marginal product of informational stock reflects the productivity improvement generated by better predictions and more informed decisions. When informational stock increases, firms can allocate resources more efficiently, reduce uncertainty in operations, and respond more effectively to changing conditions.

The marginal product of informational stock therefore captures the economic value of predictive knowledge within the production process.

4.3 Firm Optimization

Consider a competitive firm that chooses inputs to maximize profits. Let $r$denote the rental rate of capital, $w$denote the wage rate of labor, and $p_I$denote the price of informational stock. The firm's profit function can be written as

$$\Pi = F(K, L, I) - rK - wL - p_I I$$

The firm chooses input levels to maximize profits subject to the production technology.

The first-order conditions for profit maximization are

$$F_K = r$$
$$F_L = w$$
$$F_I = p_I$$

These conditions state that each input is used until its marginal product equals its price.

The third condition implies that informational stock has an equilibrium price determined by its marginal contribution to output. In other words, informational stock earns factor income just as capital and labor do.

4.4 Factor Income Shares

Under constant returns to scale, Euler's theorem implies that output can be decomposed into payments to each factor of production:

$$Y = KF_K + LF_L + IF_I$$

Substituting the equilibrium conditions derived above yields

$$Y = rK + wL + p_I I$$

This expression shows that total output is distributed among capital income, labor income, and informational income.

The share of output accruing to each factor can therefore be written as

$$\alpha = \frac{rK}{Y}$$

$$\beta = \frac{wL}{Y}$$
$$\gamma = \frac{p_I I}{Y}$$

where α, β, and γrepresent the income shares of capital, labor, and informational stock respectively.

The presence of informational stock therefore implies that economic output is distributed across three factors rather than two.

4.5 Cobb–Douglas Representation

To illustrate the implications of the model more concretely, consider a Cobb–Douglas production function that includes informational stock:

$$Y = AK^{\alpha}L^{\beta}I^{\gamma}$$

where *A*represents total factor productivity and the exponents represent factor elasticities.

Under constant returns to scale,

$$\alpha + \beta + \gamma = 1$$

In this representation, each exponent corresponds to the income share of the associated factor.

The parameter γtherefore represents the informational factor share: the proportion of output attributable to reductions in uncertainty generated by informational inputs.

This formulation makes the central argument of the paper transparent. If informational stock is a productive input but is omitted from production models, then the informational factor share must be implicitly absorbed by the remaining factors, particularly capital.

The next section examines this implication formally and demonstrates how informational productivity becomes statistically attributed to capital income when informational inputs are excluded from production estimation.

5 Informational Mispricing and the Capital Share Bias

The introduction of informational stock into the production function has direct implications for the measurement of factor income shares. In the previous section, output was shown to be distributed across three productive inputs—capital, labor, and informational stock—when informational inputs are explicitly modeled. In practice, however, most empirical production models omit informational inputs entirely. As a result, the productivity generated by informational stock must be attributed to the remaining factors of production.

This section formalizes the consequences of that omission and demonstrates that when informational inputs are productive but excluded from production estimation, their contribution is systematically attributed to capital income. The result is an upward bias in measured capital shares relative to the true structure of production.

5.1 True Production Structure

Consider a production function that includes informational stock as an independent input:

$$Y = F(K, L, I)$$

Under the assumptions introduced in the previous section, firms operating in competitive markets choose input levels such that the marginal product of each input equals its price. Let r, w, and $p_I$denote the prices of capital, labor, and informational stock respectively.

The equilibrium conditions are therefore

$$F_K = r$$
$$F_L = w$$
$$F_I = p_I$$

Under constant returns to scale, Euler's theorem implies that output can be decomposed into payments to each factor of production:

$$Y = rK + wL + p_I I$$

This expression represents the true distribution of factor income in the economy. Capital earns rK, labor earns wL, and informational stock earns $p_I I$.

5.2 Observed Production Models

In empirical work, informational inputs are rarely included explicitly in production functions. Instead, researchers typically estimate a two-factor model in which output depends only on capital and labor:

$$Y = F(K, L)$$

Within this framework, factor shares are measured using observed payments to capital and labor:

$$\alpha^{obs} = \frac{rK}{Y}$$
$$\beta^{obs} = \frac{wL}{Y}$$

Because informational inputs are not recognized as a separate factor, the informational component of output has no corresponding income category. Nevertheless, informational inputs still contribute to production. Their productivity therefore appears in the data indirectly through the returns attributed to other factors.

5.3 Absorption of Informational Productivity

To understand how this misattribution occurs, consider the Cobb–Douglas production function introduced in the previous section:

$$Y = AK^{\alpha}L^{\beta}I^{\gamma}$$

where

$$\alpha + \beta + \gamma = 1$$

In this formulation, the parameter γrepresents the share of output attributable to informational stock.

Suppose that an econometrician estimates a production function that excludes informational stock:

$$Y = \tilde{A}K^{\widehat{\alpha}}L^{\widehat{\beta}}$$

Because informational inputs remain present in the actual production process, their effects must appear somewhere in the estimated parameters. In practice, informational productivity is absorbed into the capital coefficient whenever informational stock is correlated with capital intensity.

Modern firms frequently exhibit precisely this correlation. Data infrastructure, digital platforms, and predictive systems are often embedded within capital-intensive technological environments. Firms that invest heavily in computational infrastructure, cloud systems, and digital platforms tend simultaneously to accumulate large informational stocks through data collection and predictive modeling. As a result, informational stock and capital investment frequently move together.

When informational stock and capital are positively correlated, the estimated capital elasticity reflects both the productivity of physical capital and the productivity of informational inputs embedded within capital-intensive systems.

5.4 Omitted-Variable Bias

The misattribution described above can be understood formally as a case of omitted-variable bias in production estimation.

Taking logarithms of the true Cobb–Douglas production function yields

$$\log Y = \log A + \alpha \log K + \beta \log L + \gamma \log I$$

Suppose an econometrician estimates a regression that excludes informational stock:

$$\log Y = \log \tilde{A} + \hat{\alpha} \log K + \hat{\beta} \log L + u$$

Standard omitted-variable bias results imply that the estimated capital coefficient becomes

$$\hat{\alpha} = \alpha + \gamma \frac{Cov(\log K, \log I)}{Var(\log K)}$$

Whenever informational stock is positively correlated with capital, the estimated capital elasticity exceeds the true capital elasticity.

The bias increases with both the informational factor share γ and the degree of correlation between capital and informational stock.

5.5 Implications for Measured Capital Shares

The presence of omitted informational inputs therefore produces a systematic distortion in observed factor shares.

Let the true capital share be defined as

$$\alpha = \frac{rK}{Y}$$

and the informational share be defined as

$$\gamma = \frac{p_I I}{Y}$$

When informational inputs are omitted from measurement, the observed capital share becomes

$$\alpha^{obs} = \alpha + \gamma$$

In other words, the informational share of output becomes embedded within capital income.

This mechanism implies that measured capital shares will overstate the true marginal contribution of capital whenever informational inputs play a significant productive role.

5.6 Interpretation

The result derived above has a straightforward economic interpretation. Informational stock improves productivity by enabling firms to make better decisions about pricing, allocation, logistics, and risk management. These improvements generate additional output even when the quantities of capital and labor remain unchanged.

If production models fail to account for informational inputs explicitly, the additional output generated by informational stock must be attributed to other factors. Because informational systems are often implemented through capital-intensive infrastructure, the productivity improvements they generate are frequently attributed to capital.

As a result, modern estimates of capital productivity may reflect not only the productive contribution of physical capital but also the productivity generated by informational inputs derived from observational data.

This mechanism provides a potential explanation for the rising capital share observed in many advanced economies. As firms increasingly rely on data-driven systems and predictive models, the productive role of informational inputs may expand even though production models continue to recognize only capital and labor.

The next section turns to the empirical challenge raised by this framework: how informational stock can be measured in practice and incorporated into production estimation.

6 Measuring Informational Stock

The theoretical framework developed in the preceding sections introduces informational stock as a productive input defined by reductions in uncertainty about economically relevant outcomes. While this concept is grounded in information theory, it must ultimately be connected to observable quantities if it is to play a role in empirical economic analysis. The purpose of this section is therefore to outline a practical approach to measuring informational stock within production systems.

The central insight underlying this measurement approach is that informational stock manifests itself through improvements in predictive performance. When observational datasets contain useful information about an outcome variable, predictive models trained on those datasets reduce uncertainty about that outcome. The resulting improvement in predictive accuracy provides a measurable proxy for informational value.

6.1 Predictive Models and Economic Outcomes

In many modern firms, production decisions are guided by predictive models that estimate the likelihood of economically relevant events. Examples include forecasting product demand, predicting transportation delays, estimating the probability of insurance claims, and identifying fraudulent transactions. In each case, firms rely on observational data to generate predictions about uncertain outcomes.

Let Y denote an outcome variable relevant to production. This variable may represent demand for a product, the arrival time of a shipment, the likelihood of a transaction, or any other event that influences operational decisions. A firm seeks to predict the value of Y in order to allocate resources efficiently.

Let X denote a set of observational variables available to the firm. These observations may include past transactions, user behavior, environmental conditions, or other signals generated through economic activity. The firm uses these observations to construct a predictive model that estimates the distribution of Y.

If the observations contain useful information about the outcome variable, then predictions generated using X will be more accurate than predictions based on baseline assumptions alone.

6.2 Predictive Loss and Uncertainty Reduction

Predictive accuracy is commonly evaluated using loss functions that measure the difference between predicted outcomes and realized outcomes. One widely used metric is log loss, which evaluates the accuracy of probabilistic predictions.

Let L_0 denote the predictive loss associated with a baseline model that does not incorporate observational data. Let L_1 denote the predictive loss associated with a model that incorporates observational variables X.

If the observations contain useful information about the outcome variable, then the predictive loss of the second model will be lower than that of the baseline model. The difference between these two quantities therefore represents the predictive improvement generated by the observations:

$$\Delta L = L_0 - L_1$$

When log loss is used as the evaluation metric, this improvement corresponds directly to a reduction in entropy. In other words, the reduction in predictive loss reflects the amount by which the observational variables reduce uncertainty about the outcome variable.

This reduction in uncertainty provides a practical empirical proxy for informational stock.

6.3 Informational Stock as Predictive Improvement

The previous section defined informational stock as the mutual information between observational variables and outcome variables. In empirical applications, mutual information can often be approximated using improvements in predictive performance.

Formally, informational stock can therefore be approximated as

$$I \approx L_0 - L_1$$

where the loss values correspond to predictive models estimated with and without the relevant observational variables.

This representation connects the information-theoretic concept of mutual information directly to observable quantities produced by predictive models. When a dataset substantially improves predictive performance, it contains a large amount of informational stock. When predictive improvements are small, informational stock is correspondingly limited.

Because modern firms increasingly rely on predictive systems to guide operational decisions, the informational stock generated by these systems can often be measured through their effect on forecasting accuracy.

6.4 Informational Contributions of Individual Observations

In many applications, informational stock arises from large collections of individual observations. For example, a predictive model may be trained on millions of past transactions or behavioral signals generated by users of a digital platform. Each observation contributes a small amount of information about the outcome variable.

One method for evaluating the contribution of individual observations is the Shapley value, a concept from cooperative game theory that assigns credit to individual contributors based on their marginal contributions across all possible subsets of observations. When applied to predictive systems, Shapley methods can estimate the degree to which particular observations improve predictive accuracy.

While computing exact Shapley values for large datasets may be computationally intensive, approximation techniques can provide useful estimates of informational contributions across groups of observations.

These techniques allow researchers to examine how informational stock is distributed across datasets and how different types of observations contribute to predictive performance.

6.5 Linking Informational Stock to Economic Output

Once informational stock has been measured through predictive improvements, the next step is to link informational inputs to economic output. This connection arises through the effect of predictions on operational decisions.

Improved predictions allow firms to allocate resources more efficiently. For example, accurate demand forecasts allow retailers to stock the correct quantity of goods at each location. Predictive routing systems allow logistics firms to reduce delivery times and fuel consumption. Risk models allow financial institutions to allocate capital more effectively.

In each case, predictive improvements translate into measurable economic benefits such as increased revenue, reduced costs, or improved asset utilization. These benefits represent the economic value generated by informational stock within the production process.

From the perspective of the production function introduced earlier, informational stock increases output by improving the productivity of capital and labor inputs. When predictions become more accurate, the same physical resources can generate greater output.

6.6 Practical Estimation

In practice, estimating informational stock requires several steps. First, the relevant outcome variables must be identified. These outcomes correspond to events or quantities that influence production decisions. Second, baseline predictive models must be constructed that capture the level of uncertainty present in the absence of observational data. Third, predictive models incorporating observational datasets must be estimated. Finally, the improvement in predictive performance between these models provides a measure of informational stock.

Although this process requires detailed data and modeling, the underlying logic is straightforward. Informational stock is measured by the degree to which observations improve predictions about economically relevant outcomes.

This measurement framework provides a bridge between the theoretical model introduced earlier and empirical analysis of informational inputs in modern firms.

The next section applies this framework to several industry contexts in order to illustrate the potential magnitude of informational productivity in contemporary production systems.

7 Industry Illustrations and Informational Productivity

The theoretical framework developed in the preceding sections suggests that informational stock may represent a substantial productive input in modern firms. The purpose of this section is to illustrate the potential magnitude of informational productivity using stylized examples drawn from several industries in which predictive systems and behavioral data play a central operational role.

These examples are not intended as precise empirical estimates. Rather, they serve to demonstrate how informational inputs derived from observational data can contribute significantly to firm output. In each case, the analysis considers the role of predictive systems in improving operational decisions and evaluates how informational stock may influence production outcomes.

7.1 Mobility Platforms

Mobility platforms provide a clear example of how informational inputs derived from behavioral data can influence production efficiency. These platforms match passengers with drivers in real time using predictive algorithms that incorporate large volumes of observational data, including historical ride patterns, geographic demand fluctuations, traffic conditions, and user behavior.

Without predictive systems, the allocation of drivers to passengers would be substantially less efficient. Drivers might remain idle while passengers wait for service, and vehicles might be located far from areas of high demand. Predictive algorithms allow the platform to anticipate where demand will arise and position drivers accordingly. These predictions reduce waiting times, increase vehicle utilization, and improve overall service efficiency.

From the perspective of the production framework introduced earlier, informational stock in this context consists of the predictive knowledge embedded in the platform’s data systems. The observational variables include millions of past ride transactions, location signals, and behavioral patterns. When these observations are processed through predictive models, they reduce uncertainty about where and when ride requests will occur.

The resulting predictions improve matching efficiency between drivers and passengers, which increases the number of successful rides per unit of time. In effect, informational stock increases the productivity of both labor (drivers) and capital (vehicles and platform infrastructure).

To illustrate the potential magnitude of informational productivity, consider a mobility platform with a market valuation of approximately $145 billion and roughly 200 million users. Suppose that a portion of this value reflects the predictive systems that allow the platform to coordinate supply and demand efficiently. If informational inputs account for even twenty percent of the platform's productive capacity, the implied informational factor share would correspond to roughly $29 billion in economic value.

While this estimate is purely illustrative, it demonstrates how predictive systems derived from large-scale observational data may contribute substantially to the productivity of platform-based firms.

7.2 Retail Logistics and Demand Forecasting

Retail firms provide another environment in which informational stock plays a central operational role. Large retailers maintain extensive datasets describing consumer purchasing patterns, seasonal trends, promotional responses, and geographic demand variation. These datasets are used to construct demand forecasting systems that guide inventory management, pricing decisions, and supply chain coordination.

Accurate demand forecasts allow retailers to allocate inventory more efficiently across stores and distribution centers. When predictions are accurate, firms can avoid both stockouts and excess inventory. Reducing these inefficiencies improves both revenue and cost performance.

In the absence of detailed observational data, retailers would face greater uncertainty about consumer demand. Inventory allocation would rely more heavily on coarse heuristics or historical averages. These approaches often produce significant inefficiencies because they fail to account for local variation in demand patterns.

Informational stock generated through demand forecasting systems reduces this uncertainty. Observational data allows firms to predict which products will sell in particular locations at particular times. These predictions guide inventory allocation and logistics planning throughout the supply chain.

The resulting improvements in inventory management can generate substantial economic value. Even small improvements in forecasting accuracy can translate into large financial gains when applied across thousands of products and hundreds of retail locations.

If predictive systems reduce forecasting error by a measurable margin, the resulting efficiency gains represent the economic output generated by informational stock within the retail production system.

7.3 Airline Operations and Revenue Management

Airline operations offer another example of informational inputs influencing production outcomes. Airlines rely heavily on predictive systems to manage pricing, seat allocation, and route planning. These systems analyze historical booking patterns, seasonal demand fluctuations, and passenger behavior in order to forecast demand for individual flights.

Revenue management systems use these forecasts to determine ticket prices and seat availability across different fare classes. By adjusting prices dynamically in response to predicted demand, airlines can increase load factors and maximize revenue per flight.

Operational decisions within airlines also rely on predictive information. Maintenance schedules, crew assignments, and route planning all incorporate forecasts of passenger demand and operational conditions. These forecasts help airlines allocate resources efficiently across a complex network of flights and destinations.

In this context, informational stock consists of the predictive knowledge embedded in airline forecasting systems. Observational data from past bookings and travel patterns reduces uncertainty about future demand, allowing airlines to make more informed decisions about pricing and capacity allocation.

Even modest improvements in demand prediction can generate substantial economic value in this industry. Airlines operate with thin margins and large fixed costs, so improvements in load factors or pricing efficiency can significantly affect profitability.

7.4 Informational Productivity Across Industries

The examples discussed above illustrate a common pattern across modern industries. Firms collect large volumes of observational data describing economic activity. These observations are transformed into predictive knowledge through statistical modeling and algorithmic systems. The resulting predictions guide operational decisions that improve the allocation of capital and labor.

In each case, productivity improvements arise because informational systems reduce uncertainty about economically relevant outcomes. The resulting predictions enable firms to operate more efficiently than would be possible in the absence of these informational inputs.

While the magnitude of informational productivity varies across industries, the mechanism is broadly similar. Observational data reduces uncertainty, predictive systems generate forecasts, and these forecasts improve operational decisions.

From the perspective of the production framework developed earlier, these improvements correspond to increases in informational stock. Informational stock enhances the productivity of capital and labor by enabling more accurate and efficient resource allocation.

7.5 Market-Level Implications

The industry examples discussed above suggest that informational inputs may account for a substantial share of modern firm productivity. When aggregated across the economy, the potential magnitude of informational productivity becomes even more significant.

Publicly traded firms in the United States collectively represent tens of trillions of dollars in market capitalization. If a meaningful portion of this value reflects productivity generated by informational systems built on observational data, then the aggregate informational factor share of the economy could be extremely large.

For example, suppose that informational inputs account for fifteen to twenty-five percent of output in industries heavily dependent on predictive systems. If similar dynamics apply across a broad range of sectors, the aggregate informational share of economic output could plausibly reach several trillion dollars.

Such estimates remain speculative, but they highlight the potential scale of informational productivity in modern economies. If informational inputs represent a substantial productive factor that is not explicitly recognized in production models, the resulting mismeasurement of factor income shares could be correspondingly large.

8 Informational Stock and National Income Accounting

The framework developed in this paper has implications not only for firm-level production but also for the measurement of economic activity at the macroeconomic level. National income accounting systems and growth accounting frameworks typically decompose economic output into contributions from capital, labor, and technological progress. If informational stock constitutes a productive input that is not explicitly recognized in these systems, then a portion of modern economic activity may be systematically mismeasured.

This section examines how informational inputs influence the measurement of economic growth and factor income shares. The analysis suggests that informational stock may currently be embedded within both capital income and the residual category of total factor productivity.

8.1 Standard Growth Accounting

Growth accounting frameworks decompose changes in aggregate output into contributions from capital accumulation, labor input, and technological progress. In its standard form, the growth rate of output is written as

$$\frac{\dot{Y}}{Y} = \alpha\frac{\dot{K}}{K} + \beta\frac{\dot{L}}{L} + \frac{\dot{A}}{A}$$

where Arepresents total factor productivity and the parameters $\alpha$and βcorrespond to the income shares of capital and labor.

Within this framework, increases in output that cannot be explained by capital accumulation or labor growth are attributed to technological progress. The residual productivity term therefore captures improvements in efficiency that are not directly associated with observable inputs.

However, if informational stock is a productive input that influences output, the correct growth decomposition should include informational accumulation as a separate component.

8.2 Informational Growth Accounting

When informational stock is included explicitly, the production function becomes

$$Y = F(K, L, I)$$

and the corresponding growth decomposition can be written as

$$\frac{\dot{Y}}{Y} = \alpha\frac{\dot{K}}{K} + \beta\frac{\dot{L}}{L} + \gamma\frac{\dot{I}}{I} + \frac{\dot{A}}{A}$$

where γrepresents the informational factor share.

In this formulation, growth in informational stock contributes directly to output growth. Improvements in predictive systems, data accumulation, and informational infrastructure increase I, which in turn increases output even when the quantities of capital and labor remain unchanged.

If informational inputs are omitted from the growth accounting framework, the term

$$\gamma\frac{\dot{I}}{I}$$

must be absorbed elsewhere in the decomposition. In practice, this contribution may appear as either capital deepening or unexplained technological progress.

8.3 Informational Capital Deepening

Modern firms frequently invest in infrastructure designed to generate and process observational data. These investments include data storage systems, cloud computing resources, machine learning infrastructure, and digital platforms capable of collecting large volumes of behavioral data.

Although these systems are often classified as capital investments in accounting frameworks, their productive value frequently derives from the informational stock they generate rather than from the physical infrastructure itself. The economic benefits arise when observational data collected through these systems improves predictions and decision-making.

As a result, informational productivity may appear in empirical data as increased returns to capital. Capital investments that support data collection and predictive systems may generate productivity improvements that are actually driven by informational inputs embedded within those systems.

This dynamic contributes to the upward bias in measured capital shares discussed earlier in the paper.

8.4 Informational Inputs and the Productivity Residual

Informational inputs may also appear within the residual component of growth accounting frameworks. Total factor productivity is often interpreted as a measure of technological progress or efficiency improvements that cannot be explained by observable inputs.

However, many productivity improvements in modern industries arise from better predictions and more efficient coordination enabled by informational systems. Demand forecasting, algorithmic pricing, logistics optimization, and predictive risk management all reduce inefficiencies within economic systems.

If these informational improvements are not explicitly modeled as productive inputs, their effects will appear in the productivity residual. In this sense, a portion of measured total factor productivity may reflect informational accumulation rather than purely technological change.

Recognizing informational stock as a separate factor of production therefore offers a potential explanation for some components of the productivity residual observed in empirical growth studies.

8.5 Economy-Wide Implications

The macroeconomic implications of informational productivity depend on the scale at which informational inputs influence production across the economy. In sectors where predictive systems and observational datasets play a central operational role, informational inputs may account for a substantial share of output.

Digital platforms, logistics networks, financial services, retail systems, and mobility services all rely heavily on predictive models built from observational data. As these systems expand, the informational stock embedded within them grows as well.

If informational productivity constitutes even a modest fraction of output across these sectors, the aggregate informational share of economic production could be extremely large. In such a scenario, the omission of informational inputs from national accounting frameworks would produce significant distortions in measured factor shares.

In particular, the capital share of income would appear larger than its true value because informational productivity embedded within capital-intensive systems would be attributed to capital.

8.6 Implications for Economic Measurement

Recognizing informational stock as a productive input suggests that economic measurement systems may need to evolve in order to capture the informational structure of modern production. Traditional accounting frameworks were developed in an era when physical capital and labor dominated the production process. As production becomes increasingly dependent on informational systems, new measurement approaches may be required.

Incorporating informational stock into economic measurement would involve identifying the observational datasets and predictive systems that reduce uncertainty about economically relevant outcomes. Improvements in predictive accuracy could then be linked to operational performance and output growth.

Such an approach would allow informational inputs to be measured directly rather than being absorbed into capital returns or productivity residuals.

9 Objections and Identification Challenges

The framework developed in this paper introduces informational stock as an independent factor of production. Because this claim has implications for the interpretation of capital income shares and productivity growth, it raises several potential objections. This section considers the most important conceptual and empirical challenges associated with identifying informational inputs and measuring their economic contribution.

The purpose of this discussion is not to claim that informational stock can be measured perfectly in all contexts. Rather, it is to demonstrate that the omission of informational inputs from production models represents a systematic limitation in existing frameworks and that credible empirical strategies exist for addressing this limitation.

9.1 Informational Inputs as Capital

One possible objection is that informational systems should already be treated as a form of capital. Firms often invest in data infrastructure, computational systems, and digital platforms that support the collection and analysis of observational data. These investments appear in corporate accounts as capital expenditures and therefore might be interpreted as part of the capital input in production.

While it is true that informational systems frequently require capital infrastructure, the productive role of informational stock differs fundamentally from that of physical capital. Physical capital contributes to production through its physical or mechanical capabilities. Machines produce goods, vehicles transport materials, and buildings provide space for economic activity.

Informational stock contributes to production through a different mechanism. Its value arises from reductions in uncertainty about economically relevant outcomes. Informational inputs improve decision-making rather than directly transforming materials or providing physical capacity.

The distinction becomes clear when considering how informational systems generate productivity gains. Improvements in predictive models can increase output even when the quantity of physical capital remains unchanged. For example, better demand forecasts may allow a retailer to allocate existing inventory more efficiently, increasing sales without increasing the amount of physical capital employed.

For this reason, informational stock should be treated as conceptually distinct from physical capital even when the infrastructure used to generate information is capital-intensive.

9.2 Informational Inputs as Technology

A second objection is that informational productivity may simply reflect technological progress. In standard growth accounting frameworks, improvements in efficiency that cannot be explained by capital or labor are captured by the technology term or total factor productivity. Informational improvements might therefore be interpreted as part of technological change rather than as a separate factor of production.

While informational systems certainly rely on technological advances, the productivity improvements generated by informational inputs arise from the accumulation of observational data rather than from the invention of new production technologies alone. Predictive models improve because they are trained on large datasets describing economic behavior. As more observations become available, the informational stock embedded in these models increases.

This dynamic differs from the typical interpretation of technological progress. Technological innovations often improve production capabilities independently of the quantity of observational data available. Informational stock, by contrast, grows through the accumulation and analysis of observations about economic activity.

Recognizing informational stock as a separate factor therefore allows economists to distinguish between productivity improvements generated by technological innovation and those generated by the accumulation of predictive knowledge derived from observational data.

9.3 Endogeneity of Informational Inputs

Another challenge arises from the potential endogeneity of informational inputs. Firms that invest heavily in capital infrastructure may also invest heavily in data collection and predictive systems. If informational stock is correlated with capital intensity, empirical attempts to estimate informational productivity may encounter identification problems.

This issue is closely related to the omitted-variable bias mechanism discussed earlier in the paper. When informational inputs are correlated with capital, the estimated capital elasticity absorbs part of the informational factor share. Separating these effects empirically requires independent measures of informational stock.

The measurement framework proposed in Section 6 offers one approach to addressing this challenge. Informational stock can be proxied using improvements in predictive performance generated by observational datasets. Because predictive improvements can often be measured directly within statistical models, they provide a way to quantify informational inputs independently of capital investments.

Although empirical implementation may require detailed data and careful econometric design, the presence of endogeneity does not invalidate the conceptual framework. Rather, it highlights the importance of developing better measurement tools for informational inputs.

9.4 Measurement Error

A further objection concerns the potential difficulty of measuring informational stock accurately. Predictive systems are complex, and the relationship between predictive improvements and economic output may vary across contexts. Estimating informational productivity therefore requires detailed knowledge of the operational systems used by firms.

While these challenges are real, they are not unique to informational inputs. Economists routinely estimate the productivity of capital, labor quality, and intangible assets despite substantial measurement challenges. The presence of measurement difficulties does not imply that the underlying economic variables do not exist.

Moreover, advances in data science and machine learning have made predictive performance metrics increasingly accessible. Firms routinely track forecasting accuracy, model performance, and operational outcomes associated with predictive systems. These metrics provide a starting point for empirical measurement of informational stock.

9.5 Non-Rivalry and Scale Effects

Another issue concerns the non-rivalrous nature of informational inputs. Unlike physical capital, informational stock can often be used simultaneously across many production processes. A predictive model developed for one application may generate value across millions of transactions or decisions.

This property implies that informational productivity may exhibit strong scale effects. When informational systems are applied across large datasets or large networks of users, small improvements in predictive accuracy can produce substantial aggregate gains.

While these scale effects may complicate measurement, they are consistent with the conceptual framework developed earlier in the paper. Non-rivalry is a defining characteristic of informational inputs and helps explain why informational productivity may be particularly large in digital and platform-based industries.

9.6 Conceptual Summary

The objections discussed above highlight several important challenges associated with incorporating informational inputs into production models. Informational systems are closely related to capital infrastructure, technological progress, and large-scale data aggregation. These relationships can complicate empirical identification.

Nevertheless, the central claim of the paper remains robust: informational stock represents a measurable reduction in uncertainty about economically relevant outcomes, and this reduction in uncertainty improves economic productivity. When informational inputs are productive but omitted from production models, their contribution must appear elsewhere in measured factor returns.

The presence of measurement challenges does not eliminate the economic significance of informational inputs. Instead, it underscores the need for new theoretical and empirical frameworks capable of capturing the informational structure of modern production systems.

10 Conclusion

This paper has proposed a framework for incorporating informational inputs directly into economic production theory. The central argument is that modern firms increasingly rely on structured observations of economic activity to reduce uncertainty about economically relevant outcomes. These reductions in uncertainty improve predictions and allow firms to allocate resources more efficiently. The resulting predictive knowledge—referred to in this paper as informational stock—functions as a productive input alongside capital and labor.

The framework introduced here draws on concepts from information theory to provide a formal definition of informational stock. Specifically, informational stock is defined as the mutual information between observational variables and economically relevant outcome variables. Mutual information measures the degree to which observations reduce uncertainty about outcomes. When observational datasets improve predictions about demand, risk, logistics, or market behavior, they generate informational stock.

Incorporating informational stock into a production function yields a simple but important implication. If informational inputs contribute to output but are not explicitly recognized in production models, their productivity must be attributed to other factors of production. Because informational systems are frequently embedded within capital-intensive technological environments, the productivity generated by informational inputs is often attributed to capital.

This mechanism implies that observed capital income shares may overstate the true marginal contribution of capital in modern economies. When informational inputs are omitted from production estimation, the informational share of output becomes embedded within capital income. The resulting distortion can be understood formally as a case of omitted-variable bias in production estimation.

The potential magnitude of this distortion may be substantial. Modern firms increasingly rely on predictive systems built from large-scale observational datasets. Mobility platforms coordinate millions of transportation transactions using predictive algorithms. Retailers allocate inventory across vast logistics networks using demand forecasting systems. Financial institutions evaluate credit risk and market behavior using predictive models trained on extensive historical datasets. In each of these cases, productivity improvements arise because informational systems reduce uncertainty and improve decision-making.

Industry illustrations presented in this paper suggest that informational inputs may account for a significant share of productivity in sectors that rely heavily on predictive systems. When aggregated across the broader economy, the resulting informational factor share could represent a substantial portion of total economic output.

These observations suggest that informational factor mispricing may represent one of the largest structural distortions in contemporary economic measurement. If informational productivity is currently embedded within capital income and productivity residuals, then the true structure of production in modern economies may differ significantly from the structure implied by conventional two-factor models.

Recognizing informational stock as a productive input has several implications for future research. First, it suggests that empirical production models may benefit from incorporating measures of predictive knowledge derived from observational datasets. Advances in data science and machine learning provide tools for quantifying improvements in predictive accuracy, which may serve as proxies for informational stock.

Second, incorporating informational inputs into growth accounting frameworks may help clarify the sources of productivity improvements in modern economies. Some components of measured total factor productivity may reflect the accumulation of predictive knowledge rather than purely technological innovation.

Third, the informational production framework highlights the growing economic importance of observational data generated through human activity. As digital systems increasingly collect and analyze behavioral signals, informational stock derived from these observations becomes a central component of modern production systems.

More broadly, the framework developed in this paper suggests that economic theory may need to evolve in order to capture the informational structure of modern production. Traditional models were developed in an era when physical capital and human labor dominated the production process. In contemporary economies, predictive knowledge derived from observational data plays an increasingly central role in coordinating economic activity.

Recognizing informational stock as a factor of production provides a first step toward understanding this transformation. By linking information theory with production economics, the framework introduced here offers a way to measure and analyze the informational inputs that increasingly shape modern economic systems.

Future work may extend this framework by developing empirical estimates of informational stock across industries and by exploring how informational inputs interact with capital and labor in complex production environments. Such work may ultimately help clarify the role of information in shaping economic growth, productivity, and the distribution of income in the modern economy.

Appendix A

Mathematical Derivations

This appendix provides formal derivations supporting the theoretical results presented in the main text. In particular, it derives the factor decomposition of output when informational stock is included as a productive input and demonstrates the conditions under which informational productivity is absorbed into measured capital income when informational inputs are omitted from production estimation.

A.1 Factor Decomposition with Informational Stock

Consider the production function introduced in the main text:

$$Y = F(K, L, I)$$

where K denotes capital, L denotes labor, and I denotes informational stock. The production function is assumed to be continuously differentiable and to exhibit constant returns to scale.

Under constant returns to scale, the production function satisfies the property

$$F(\lambda K, \lambda L, \lambda I) = \lambda F(K, L, I)$$

for any positive scalar λ.

Euler's theorem for homogeneous functions implies that

$$F(K, L, I) = KF_K + LF_L + IF_I$$

where F_K, F_L, and F_I denote the marginal products of capital, labor, and informational stock respectively.

Under competitive conditions, each factor is paid its marginal product. Let r, w, and p_I denote the prices of capital, labor, and informational stock. Profit maximization implies

$$F_K = r$$
$$F_L = w$$
$$F_I = p_I$$

Substituting these expressions into the Euler decomposition yields

$$Y = rK + wL + p_I I$$

This expression represents the true distribution of factor income when informational inputs are recognized as a productive factor.

Dividing both sides by output gives the factor share decomposition

$$1 = \frac{rK}{Y} + \frac{wL}{Y} + \frac{p_I I}{Y}$$

Define the capital share, labor share, and informational share as

$$\alpha = \frac{rK}{Y}$$
$$\beta = \frac{wL}{Y}$$
$$\gamma = \frac{p_I I}{Y}$$

The factor share identity therefore becomes

$$\alpha + \beta + \gamma = 1$$

This expression extends the traditional two-factor income decomposition to include informational inputs.

A.2 Cobb–Douglas Representation

To illustrate the structure of informational production more explicitly, consider the Cobb–Douglas production function

$$Y = AK^{\alpha} L^{\beta} I^{\gamma}$$

where Arepresents total factor productivity and the parameters α, β, and γrepresent output elasticities.

Under constant returns to scale,

$$\alpha + \beta + \gamma = 1$$

The marginal products of each input are

$$F_K = \alpha A K^{\alpha-1} L^{\beta} I^{\gamma}$$
$$F_L = \beta A K^{\alpha} L^{\beta-1} I^{\gamma}$$
$$F_I = \gamma A K^{\alpha} L^{\beta} I^{\gamma-1}$$

Multiplying each marginal product by the corresponding input yields

$$KF_K = \alpha Y$$
$$LF_L = \beta Y$$
$$IF_I = \gamma Y$$

These expressions confirm that the parameters of the Cobb–Douglas function correspond directly to factor income shares.

A.3 Omitted Variable Bias

Suppose the true production technology is

$$Y = AK^{\alpha}L^{\beta}I^{\gamma}$$

Taking logarithms yields

$$\log Y = \log A + \alpha \log K + \beta \log L + \gamma \log I$$

Now suppose that informational stock is omitted from the estimated production function. The econometrician estimates

$$\log Y = \log \tilde{A} + \hat{\alpha} \log K + \hat{\beta} \log L + u$$

The omitted informational input appears in the error term

$$u = \gamma \log I + \varepsilon$$

Standard omitted-variable bias results imply that the estimated capital coefficient becomes

$$\hat{\alpha} = \alpha + \gamma \frac{Cov(\log K, \log I)}{Var(\log K)}$$

Whenever informational stock is positively correlated with capital, the estimated capital elasticity exceeds the true elasticity.

A.4 Absorption of Informational Factor Share

Consider the limiting case in which informational stock scales proportionally with capital investment across firms. In this case

$$\log I = \log K + c$$

for some constant c.

Substituting this expression into the true production function yields

$$Y = AK^{\alpha+\gamma}L^{\beta}e^{\gamma c}$$

In this scenario, the informational factor share becomes statistically indistinguishable from capital productivity. The estimated capital elasticity becomes

$$\hat{\alpha} = \alpha + \gamma$$

This result shows that when informational inputs scale with capital investment, the informational share of output becomes embedded within measured capital returns.

A.5 Interpretation

The derivations above demonstrate that informational stock behaves mathematically as a standard factor of production within a competitive equilibrium framework. When informational inputs are recognized explicitly, output is distributed across capital, labor, and informational stock.

However, when informational inputs are omitted from production estimation, their productivity must appear elsewhere in the data. Because informational systems are often embedded within capital-intensive infrastructures, the productivity generated by informational inputs is frequently attributed to capital.

This mechanism provides a theoretical explanation for how informational productivity can inflate measured capital income shares in empirical production studies.

Appendix B

Empirical Estimation of Informational Stock

The theoretical framework developed in the main text defines informational stock as the reduction in uncertainty about economically relevant outcomes generated by structured observations. While this definition is grounded in information theory, empirical implementation requires a practical method for estimating informational inputs using observable data. This appendix outlines an approach for approximating informational stock using predictive performance metrics derived from statistical models.

The basic idea underlying this approach is straightforward. If a dataset reduces uncertainty about an outcome variable, then predictive models trained on that dataset should perform better than models that do not incorporate those observations. The improvement in predictive performance provides a measurable proxy for informational value.

B.1 Predictive Modeling Framework

Consider a predictive task in which a firm attempts to forecast an economically relevant outcome Y. Examples of such outcomes include product demand, transaction probabilities, shipment arrival times, or credit default events.

Let X denote a set of observational variables available to the firm. These observations may include past transactions, behavioral signals, environmental conditions, or other data generated through economic activity.

A predictive model uses these observations to estimate the probability distribution of the outcome variable. Formally, the model produces a predictive distribution

$$\hat{P}(Y \mid X)$$

which represents the model's estimate of the probability of each possible outcome.

B.2 Baseline Uncertainty

Before incorporating observational data, the firm faces baseline uncertainty about the outcome variable. This baseline uncertainty corresponds to the entropy of the outcome distribution

$$H(Y)$$

In practice, the baseline prediction may correspond to a simple model that relies only on aggregate averages or unconditional probabilities. Such models do not incorporate detailed observational data and therefore leave a large amount of uncertainty unresolved.

B.3 Conditional Uncertainty

When observational variables are incorporated into the predictive model, the remaining uncertainty about the outcome variable becomes the conditional entropy

$$H(Y \mid X)$$

Conditional entropy measures the uncertainty that remains after accounting for the information contained in the observations.

If the observational variables contain strong predictive signals, the conditional entropy will be substantially smaller than the baseline entropy. The difference between these quantities represents the amount of uncertainty reduction generated by the observations.

B.4 Mutual Information as Informational Stock

The informational value of the observations is therefore given by mutual information:

$$I(X;Y) = H(Y) - H(Y \mid X)$$

This quantity measures the degree to which the observational variables reduce uncertainty about the outcome variable. Within the production framework developed in the paper, this reduction in uncertainty corresponds to informational stock.

In empirical applications, mutual information can be approximated through improvements in predictive model performance.

B.5 Predictive Loss as a Practical Proxy

Many predictive systems are evaluated using loss functions that measure the accuracy of predicted probability distributions. One commonly used metric is log loss, which evaluates how closely predicted probabilities match realized outcomes.

Let L_0denote the predictive loss associated with a baseline model that does not incorporate observational data. Let L_1denote the predictive loss associated with a model that incorporates observational variables.

The improvement in predictive performance is therefore

$$\Delta L = L_0 - L_1$$

When log loss is used as the evaluation metric, this improvement corresponds directly to a reduction in entropy. As a result, predictive loss improvement provides a practical proxy for mutual information.

Informational stock can therefore be approximated as

$$I \approx L_0 - L_1$$

This relationship provides a direct empirical connection between predictive modeling and informational production.

B.6 Attribution of Informational Contributions

In many applications, informational stock arises from large datasets composed of numerous individual observations. Understanding how these observations contribute to predictive performance can be useful for analyzing the structure of informational production.

One approach to attribution uses Shapley values from cooperative game theory. The Shapley value assigns credit to individual contributors based on their marginal contribution to a collective outcome across all possible subsets of contributors.

In the context of predictive systems, Shapley values can be used to estimate the contribution of individual observations or groups of observations to predictive performance. Each observation is evaluated by measuring how much it improves predictive accuracy when added to different subsets of the dataset.

While computing exact Shapley values may be computationally intensive for very large datasets, approximate methods can provide useful estimates of informational contributions across observations.

These techniques allow researchers to analyze how informational stock is distributed across datasets and how different types of observations contribute to predictive knowledge.

B.7 Linking Informational Improvements to Output

Once informational stock has been measured through predictive improvements, the next step is to connect informational inputs to economic output.

Predictive improvements influence production through their effect on operational decisions. For example, accurate demand forecasts allow firms to allocate inventory efficiently across locations. Predictive routing systems allow logistics firms to minimize delivery times and fuel costs. Risk prediction models allow financial institutions to allocate capital more effectively.

In each case, predictive improvements translate into measurable economic benefits such as increased revenue, reduced operational costs, or improved resource utilization.

Within the production framework developed in the paper, these benefits correspond to increases in output generated by informational stock. Informational inputs improve the productivity of capital and labor by enabling more efficient allocation of resources.

B.8 Empirical Implementation

Empirical implementation of the informational production framework involves several steps. First, researchers must identify outcome variables relevant to the production process. These outcomes correspond to events or quantities that influence operational decisions.

Second, baseline predictive models must be constructed to estimate the level of uncertainty associated with these outcomes in the absence of observational data.

Third, predictive models incorporating observational datasets must be estimated. The improvement in predictive performance between the baseline model and the data-informed model provides a measure of informational stock.

Finally, the relationship between informational improvements and production outcomes can be analyzed to estimate the contribution of informational inputs to economic output.

While empirical implementation may require detailed datasets and careful econometric design, the measurement framework outlined here provides a feasible pathway for estimating informational inputs in modern production systems.